Electromagnetic

مركز الفجر
للخدمات العلمية
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Lecture (8)

Chapter (5)

"Dielectrics and Capacitance"

2nd year

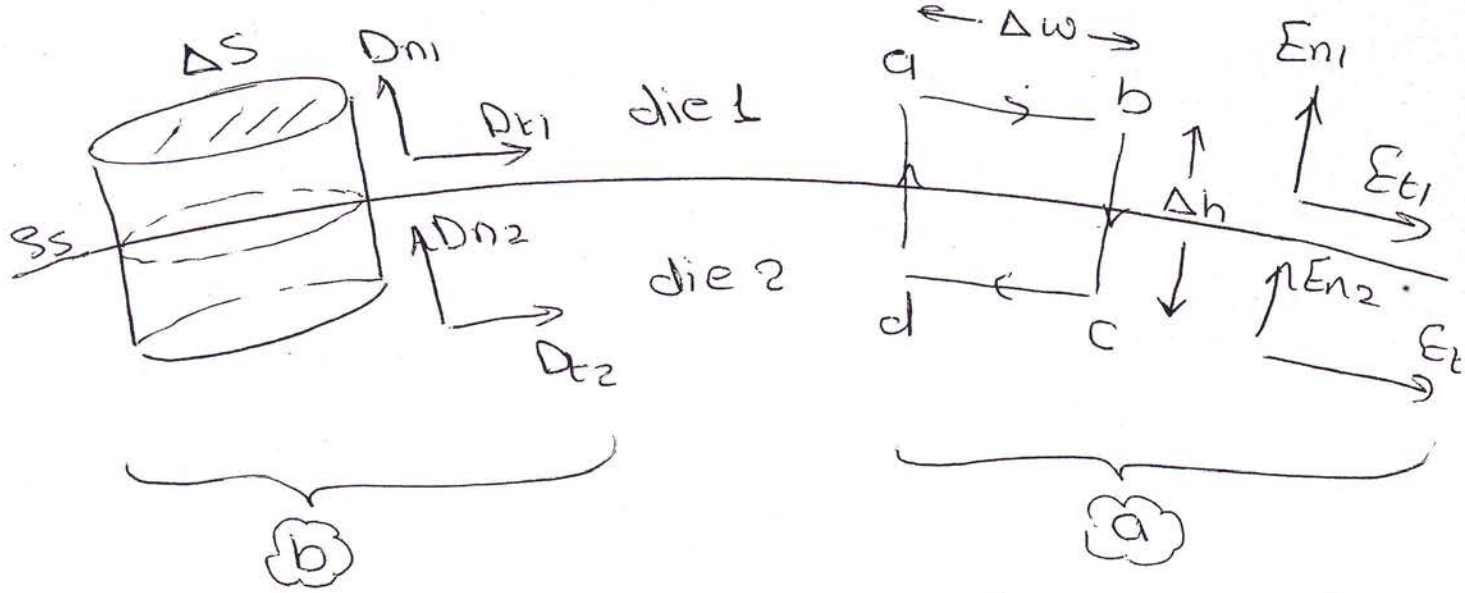
communications

Lecture (2) :

Chapter (5)

Dielectrics and Capacitance

[I] Boundary Conditions :



$\Delta w E_{t1} - \frac{1}{2}\Delta h E - \Delta w E_{t2} + \frac{1}{2}$

(a) $\oint \bar{E} \cdot d\bar{L} = 0$

Closed path is a b c d a

$$\Delta w E_{t1} - \frac{1}{2}\cancel{\Delta h E_{n1}} - \cancel{\frac{1}{2}\Delta h} E_{n2} - \Delta w E_{t2}$$

$$+ \frac{1}{2}\cancel{\Delta h} E_{n2} + \frac{1}{2}\cancel{\Delta h} E_{n1} = 0$$

$$\therefore \boxed{E_{t1} = E_{t2}}$$

(b) Gauss Law : $\oint_S \bar{D} \cdot d\bar{S} = Q_{enc}$

$$Q_{enc} = \rho_s \, \Delta S$$

$$\bar{D} \cdot \bar{S} = D_{n1} \, \Delta S \ominus D_{n2} \, \Delta S$$

← الإشارة السالبة لأن D_{n2} داخل الى السطح المغلق

$$\therefore \rho_s = D_{n1} - D_{n2}$$

$\therefore$ B.Cs :

(a) $E_{t1} = E_{t2}$.

(b) $D_{n1} - D_{n2} = \rho_s$ (if $\rho_s \rightarrow$ not given

$\therefore$ we assume $\rho_s = 0$)

$$D_{n1} = D_{n2}$$

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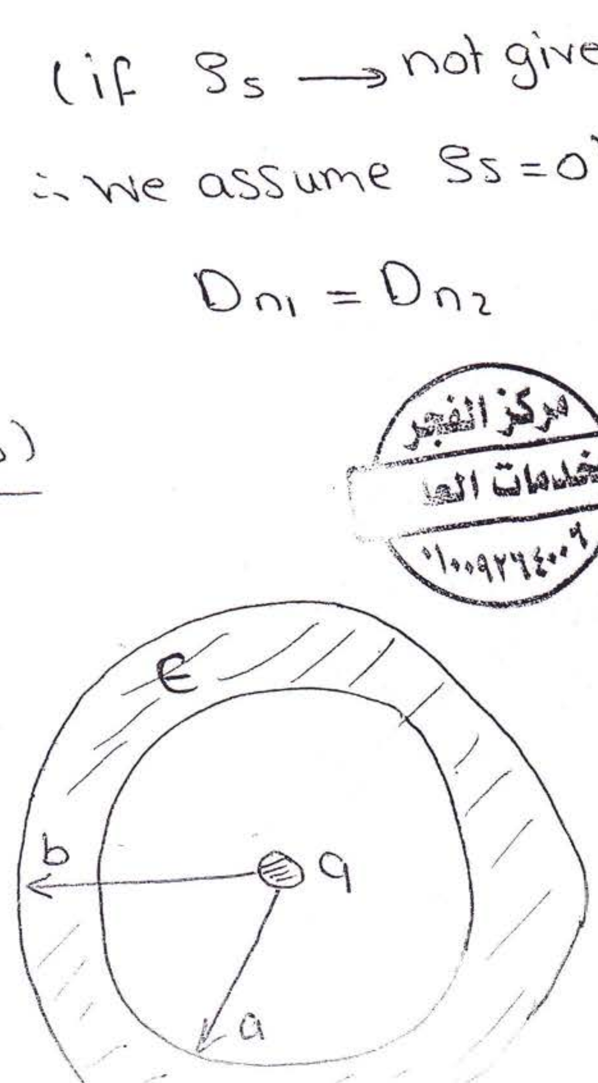
[1]

Find $\bar{E}$ and $\bar{D}$

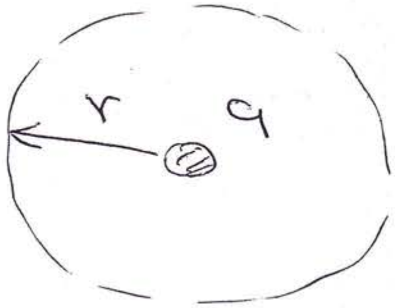
everywhere.



Solution

Using Gauss Law:

→ $r < a$:



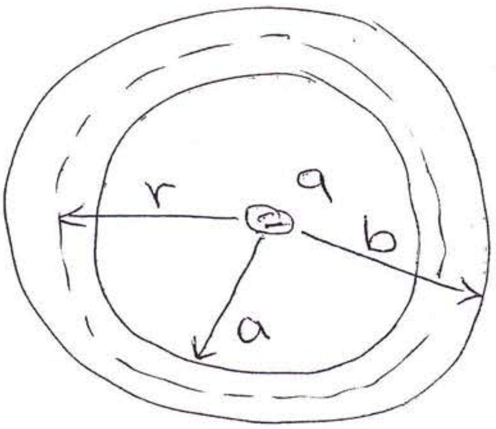
$Q_{enc} = q$

$\bar{D} \cdot \bar{S} = 4\pi r^2 D_r$

$\therefore \bar{D} = \dfrac{q}{4\pi r^2}\hat{r} \quad \bar{E} = \dfrac{q}{4\pi \epsilon_0 r^2}\hat{r}$.

→ $a < r < b$:

Can solved using Gauss or B.Cs:



$Q_{enc} = q$

$\bar{D} \cdot \bar{S} = 4\pi r^2 D_r$

$\therefore \bar{D} = \dfrac{q}{4\pi r^2}\hat{r} \quad \bar{E} = \dfrac{q}{4\pi \epsilon r^2}\hat{r}$

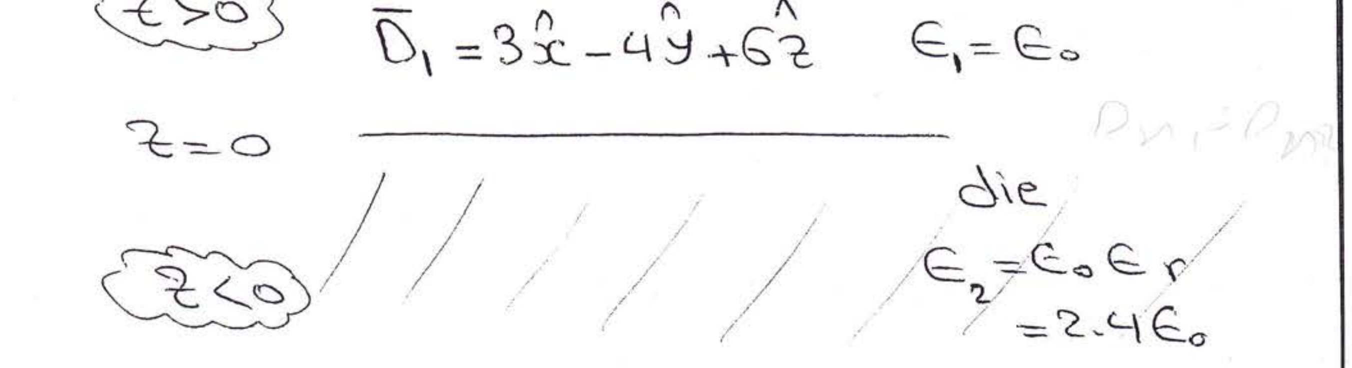
→ $r > b$: $Q_{enc} = q$

$\bar{D} = \dfrac{q}{4\pi r^2}\hat{r} \quad \bar{E} = \dfrac{q}{4\pi \epsilon_0 r^2}\hat{r}$

$$\therefore \bar{D} = \frac{q}{4\pi r^2}\hat{r} \implies$$ ملاحظة: ال $\bar{D}$ لا تعتمد على الوسط $(\text{no } \epsilon, \epsilon_0)$

$$\bar{E} = \begin{cases} \dfrac{q}{4\pi\epsilon_0 r^2}\hat{r} & r < a \\ \dfrac{q}{4\pi\epsilon_0\epsilon_r r^2}\hat{r} & a < r < b \\ \dfrac{q}{4\pi\epsilon_0 r^2}\hat{r} & r > b \end{cases}$$

$\boxed{21}$


$z>0$
$\bar{D}_1 = 3\hat{x} - 4\hat{y} + 6\hat{z}$
free spale
$\epsilon_1 = \epsilon_0$
$z=0$
$D_n = D_n$
die
$z<0$
$\epsilon_2 = \epsilon_0\epsilon_r$
$= 2.4\epsilon_0$

Find $\bar{E}_1, \bar{D}_2, \bar{E}_2$.

$\boxed{\text{Solution}}$

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$$\rightarrow \bar{E}_1 = \frac{\bar{D}_1}{\epsilon_1} = \frac{3}{\epsilon_0}\hat{x} - \frac{4}{\epsilon_0}\hat{y} + \frac{6}{\epsilon_0}\hat{z}$$

→ using B.Cs to find $\bar{E}_2, \bar{D}_2$:

⇛ normal Component: $D_{n1} = D_{n2}$ ($\hat{z}$)

($\rho_s \rightarrow$ not given)

$$D_{2z} = D_{1z} = 6$$

$$E_{2z} = \frac{D_{2z}}{\epsilon_2} = \boxed{\frac{6}{2.4\epsilon_0}}$$

⇛ tangent Components: $E_{t1} = E_{t2}$ ($\hat{x}, \hat{y}$)

$$E_{2x} = E_{1x} = \frac{3}{\epsilon_0}$$

$$E_{2y} = E_{1y} = -\frac{4}{\epsilon_0}$$

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$$\therefore \bar{E}_2 = \frac{3}{\epsilon_0}\hat{x} - \frac{4}{\epsilon_0}\hat{y} + \frac{6}{2.4\epsilon_0}\hat{z}$$

$$\bar{D}_2 = \epsilon_2 \bar{E}_2 = 2.4\epsilon_0 \bar{E}_2$$

$$= 7.2\hat{x} - 9.6\hat{y} + 6\hat{z}$$

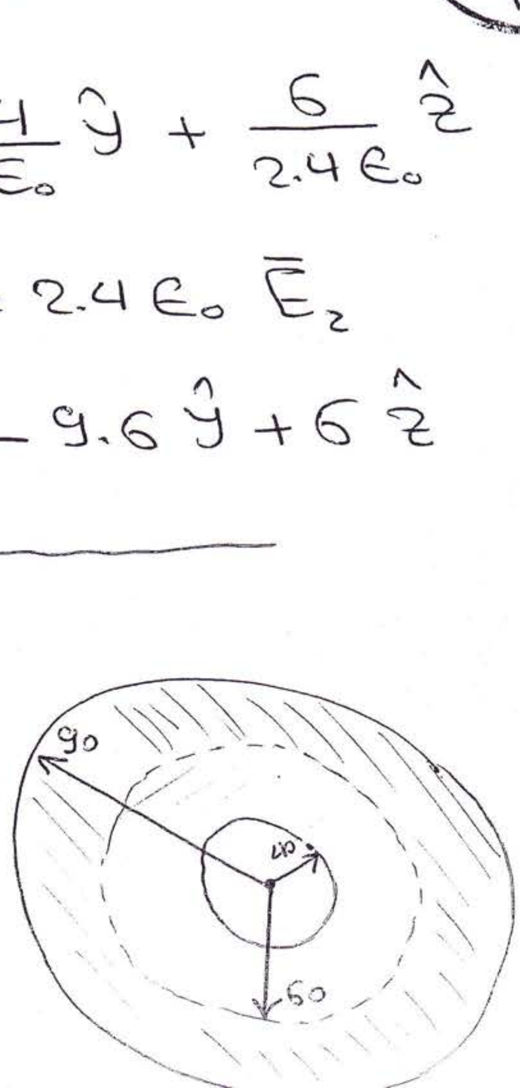
|3|.

→ $40 < r < 60$ mm:

$\epsilon_{r1} = 2.25$

→ $60 < r < 90$ mm:

$\epsilon_{r2} = 3.24$



$$\bar{E}_1 = \frac{2000}{r^2}\,\hat{r} \quad (V/m)$$

$\boxed{\text{Solution}}$

(a) Using the B.Cs:

$\Rightarrow$ normal Components: $D_{n1} = D_{n2}$ $(\hat{r})$

($\rho_s \rightarrow$ not given)

$$D_{2r} = D_{1r} = \epsilon_1 E_{1r} = \epsilon_0 \epsilon_{1r} E_{1r}$$

$$= 2.25\,\epsilon_0 \cdot \frac{2000}{r^2}$$

$$E_{2r} = \frac{D_{2r}}{\epsilon_2} = \frac{1}{3.24\,\cancel{\epsilon_0}} \cdot 2.25\,\cancel{\epsilon_0} \cdot \frac{2000}{r^2}$$

$$= \frac{1388.88}{r^2}$$

$\Rightarrow$ tangent Components: $E_{t1} = E_{t2}$ $(\hat{\theta}, \hat{\phi})$

$$E_{2\theta} = E_{1\theta} = 0$$

$$E_{2\phi} = E_{1\phi} = 0$$

$$\therefore \boxed{\bar{E}_2 = \frac{1388.88}{r^2}\,\hat{r}}$$

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(b) $W_E = \int_v \frac{\epsilon}{2} |E|^2 \, dv$

die 1 :

$$W_{E1} = \frac{\epsilon_1}{2} \int_0^{2\pi}\int_0^{\pi}\int_{0.04}^{0.06} \frac{(2000)^2}{r^4} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{(2000)^2 \epsilon_1}{2} \cdot 2\pi \cdot \frac{-1}{r}\Big|_{0.04}^{0.06} \cdot \cos\theta\Big|_{\pi}^{0}$$

$$\therefore W_{E1} = 4.17 * 10^{-3} \text{ J}$$

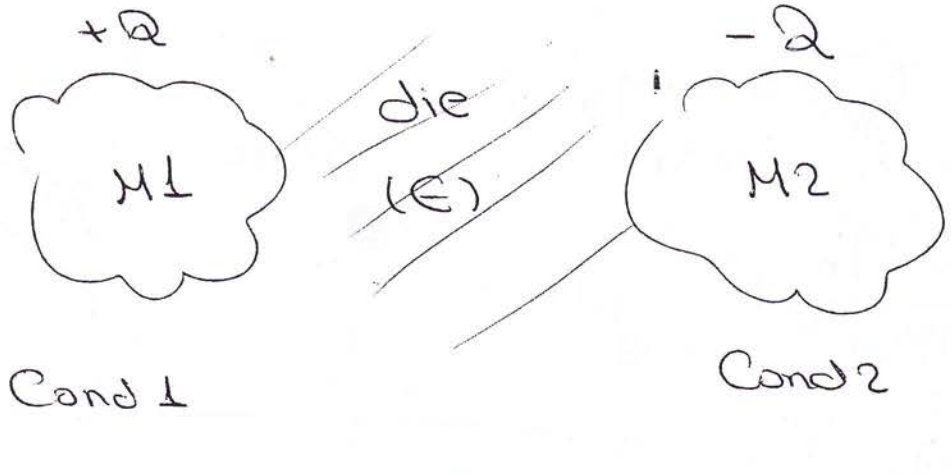
die 2 :

$$W_{E2} = \frac{\epsilon_2}{2} \int_0^{2\pi}\int_0^{\pi}\int_{0.06}^{0.09} \frac{(1389)^2}{r^4} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{(1389)^2 \epsilon_2}{2} \cdot 2\pi \cdot \frac{-1}{r}\Big|_{0.06}^{0.09} \cdot \cos\theta\Big|_{\pi}^{0}$$

$$\therefore W_{E2} = 1.93 * 10^{-3} \text{ J}$$

$\boxed{2}$ The Capacitance:



$$C = \frac{Q}{V} \quad (F)$$

V ⇒ The mag of the potential difference bet^n the 2 Conductors.

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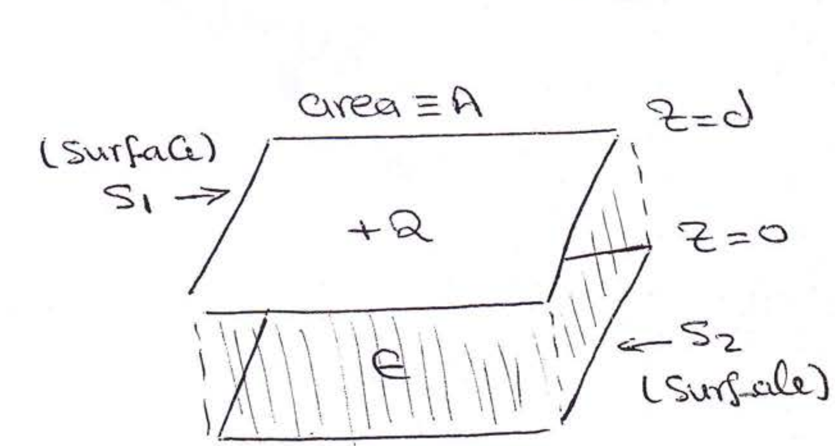
$\boxed{4}$ Find C ?



$\boxed{\text{Solution}}$

$$\bar{E} = \bar{E}_{s_1} + \bar{E}_{s_2}$$

$$= \frac{\rho_s}{2\epsilon}(-\hat{z}) + \frac{-\rho_s}{2\epsilon}\hat{z} \quad +\frac{\rho_s}{2\epsilon}\hat{z}$$

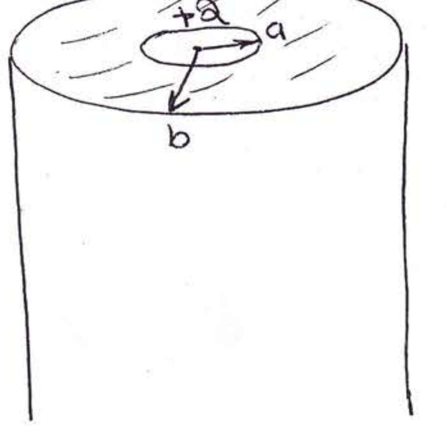
$\rho_s \hat{z}$

$$V = -\int_{-}^{+} \bar{E}\cdot d\bar{L} = -\int_{0}^{d} -\frac{\rho_s}{\epsilon}\, dz$$

$$= \frac{\rho_s}{\epsilon}\, d \qquad \frac{\rho_s}{\epsilon}$$

$$\therefore C = \frac{Q}{V} = \frac{\cancel{\rho_s}\, A}{\frac{\cancel{\rho_s}}{\epsilon}\, d} = \boxed{\frac{\epsilon A}{d}}$$

$\boxed{5}$ Find C ?



$\boxed{\text{Solution}}$

→ Find $\bar{E}$ using Gauss

↓ (مسقط أفقي)

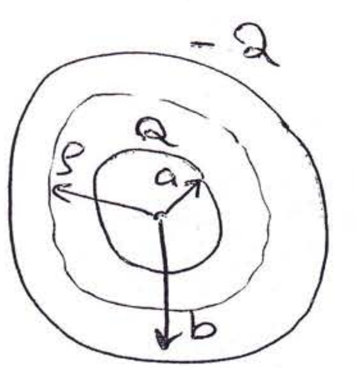
Gaussian sur (Cyl of rad ρ)

$$a < \rho < b$$

$$Q_{enc} = \bar{D}\cdot\bar{S}$$

$$Q = 2\pi\rho l D_\rho$$

$$\therefore \bar{D} = \frac{Q}{2\pi\rho l}\hat{\rho}$$



$$\bar{E} \quad \bar{D} \quad Q \quad \hat{\rho}$$

$$\therefore V = -\int_{-}^{+} \bar{E} \cdot d\bar{L}$$

$$= -\int_{b}^{a} \frac{Q}{2\pi \epsilon \rho l} \, d\rho$$

$$= -\frac{Q}{2\pi \epsilon l} \ln(\rho) \Big|_{b}^{a}$$

$$= \frac{Q}{2\pi \epsilon l} \ln\left(\frac{b}{a}\right)$$

$$\therefore C = \frac{Q}{V} = \frac{2\pi \epsilon l}{\ln(b/a)}$$

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Mansoura University
Faculty of Engineering
Communication Engineering Department

Second Year Communication
Electromagnetic Fields
Sheet (6)

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[1] A charge q C is placed at the center of a dielectric spherical shell of dielectric constant ε_r, inner radius a, and outer radius b. Find **E** and **D** everywhere.

[2] Region 1, defined by $z > 0$, is free space, while region 2, $z < 0$, is a dielectric material for which $\varepsilon_{r2} = 2.4$.

Find $\mathbf{E}_1$, $\mathbf{D}_2$, $\mathbf{E}_2$, given that $\mathbf{D}_1 = 3\,a_x - 4\,a_y + 6\,a_z$ (C/m^2).

[3] The spherical surfaces r = 40 mm and r = 90 mm are separated by two perfect dielectric shells, so that $\varepsilon_r = 2.25$ $40 \le r \le 60$ mm, and $\varepsilon_r = 3.24$ $60 \le r \le 90$ mm. If the electric field in region 1 is $\mathbf{E}_1 = (2000/r^2)\,a_r$ V/m.

(a) Find $\mathbf{E}_2$ in region 2.

(b) Find the stored energy W_E in each region.

[4] Find the capacitance of the parallel plates shown in Fig. 1.

[5] Find the capacitance of a coaxial capacitor of length L, where the inner conductor has radius a and the outer radius b. See Fig.2.

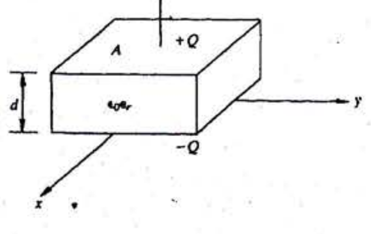

Fig. 1

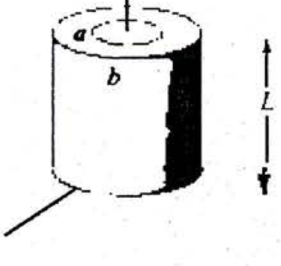

Fig. 2